The Cyclical Extraction (CE) and the Causality Test (CT) in Business Cycle Analyses: Do they complement or contradict to one another?\*

Abd Latib Talib Department of Statistics Malaysia

#### **Abstract:**

In business cycle analysis, the CE and the CT models are used during the process to select the indicators. However, the results from both models sometime were inconsistent. The aims of this paper is to examine which model as the best for business cycle analysis. The IPI is used as a benchmark indicator while the money supply M1, M2 and M3 as the tested indicators. Overall findings suggest that the outcome of the CE and CT models were very different especially in classifying the indicators as a "leading", "coincident" or "lagging" series. The findings also suggest that the CE and CT model can be used simultaneously in business cycle analysis. The CT helps to identify the relationship of each of time series while the CT provide extra information on "conformity" criterion as well as the magnitude of predictive measure.

#### Introduction

The cyclical extraction (CE) and the causality test (CT) were among the models used in business cycle analysis especially for Business Cycle Indicators (BCI) selection. However, both models have limitations. For example, the CT was unable to absorb the property of "cyclical conformity" in time series under study. Conformity is one of the main criteria in BCI selection (The Conference Board, 2000). It measures the tendency of an indicator to exhibit upswings and downswings in accordance with past business cycles (OECD, 2005). As comparison, the CE model is relatively time consuming compared with the CT model. This is because researchers need to decompose the time series step by step to eliminate its seasonal, trend and cyclical components by using different types of statistical software and methodologies.

In analyzing the "cyclical conformity" of time series, the CE model or sometime also called as growth cycle (GC) is the appropriate one. The reason is the turning points of the series can be closely compared with the turning points of general business cycles, thus, it is easier to classify whether the series under study is a "leading", "coincident" or "lagging". However, the weakness of CE model is that the decision made only based on judgement without any statistical test as compared with the CT model which adopts the statistical test in the posses of classifying the series under study. However, the CT model neglected the "cyclical conformity" criteria.

Both models have strength and weaknesses. In principle, they were supposed to support to one another. This is because the weakness of one model is offset by the strength of the other model. The question is will these two models provide the same conclusion and

<sup>\*</sup> The views and opinions expressed here reflect the author's point of view and not necessarily those of Department of Statistics Malaysia.

support to one another. Or, will these two models sometime clash to one another which implies that the results were very different between them.

The aim of this paper is to evaluate whether the CE or CT is the appropriate tools for BCI selection. For this argument, this paper will examine both models and compared the findings between them. For this purpose, the money supply be used as for the BCI and the Industrial Production Index (IPI) represents the general business cycle fluctuation.

The organisation of the paper is as follows. The following section is the literature related to the tools for business cycle analyses as well as the time series used for business cycle analysis. It will follow with the methodologies used for analysis. The findings and the concluding remarks will at the end the paper.

### 2. The Literature Review

Financial development is one of the important components in determining the economic growth. It is defined as a process that marks improvement in quantity, quality and efficiency of financial intermediary services. This process involves the interaction of economic activities and possibly associated with economic growth. There is no specific indicator to measure the financial development. However, many researchers assume that the best measurement of financial development is via financial indicators. Some use money stock usually M2 as indicators for financial development. Mohamad Yazis Ali Basah et. el (2007) and Eatzaz Ahmad & Aisha Malik (2009) used the ratio of credit value by financial intermediaries of private sectors to GDP and the ratio of Commercial bank asset to commercial bank assets plus central bank asset as financial development indicators.

Edison (2000) used M2 multiplier, the ratio of domestic credit to nominal GDP, the real interest rate on deposits, the ratio of lending-to-deposit interest rates, excess real M1 balances, and commercial bank deposits as a financial indicator. Abd. Latib Talib & Asmaddy Harris (2011) examined eighteen financial indicators including the money supply and they found that money supply is one of the best indicator that has a potential to predict in advance of the Malaysia 1997/98 financial crisis. Faiz Masnan et. Al (2013) examined the relationship between inflation, money supply, and economic growth in Malaysia, Singapore and Indonesia. They found that in Malaysia money supply does not Granger cause economic growth. Economic growth does Granger cause inflation in Malaysia, Indonesia and Singapore. Causality runs from economic growth to money supply only in Malaysia. Chimobi (2010) examined the relationship and causality between trade openness, financial development and growth in Nigeria. His findings suggest that the trade openness and financial development does have causal impact on economic growth

This paper aims to examine the relationship between the economic growth (industrial production index) money supply M1 M2 and M3 using the CE and the CT models. The CE model or the growth cycle model is a decomposition method to extract the cyclical component of time series from its long term trend. Its sometime also called the deviation

from trend's model. The long term trend is estimated using the Phase Average Trend (PAT)<sup>1</sup> and later has been improved for example by using the filtering model such as Hodrick-Prescott (HP-filter) or Christiano-Fitzgerald (CF-Filter) or Band Pass (BP-Filter). In comparing which is the best model, Nielsson and Gyomai (2011) found that the HP-filter and the CF-filter performs better than the PAT. The choice between HP-filter and CF-filter depend on the objective of analysis. If the objective is to have an early, clear and steady turning point signals then the HP-filter is the choice and use the CF filter if the analysis are sensitive to cumulative revisions. The CT model is to examine the relationship between the series in the model. Detail of the two model will be describe in the methodology section.

### 3. The Methodologies

As mention in the earlier part this paper that is to evaluate the performances of the two models for BCI selection by applying the same time series. The first method is called a decomposition method in which the time series will be decompose to its main components: seasonal, trend-cycle and irregular. The trend and cycle components are extracted using the Cyclical Extraction Method. The second method is called the Causality Test method in which to examine the relationship between the series in the model.

### 3.1 Method 1: Cyclical Extraction Methods (CE)

The CE involves three main steps. The first step is to decompose economic time series into its main components<sup>2</sup> and the second step is to extract the cyclical components and the final step is compare turning points of the series with the general business cycle turning points.

and the second step is the cyclical extraction. The third step is to establish the reference cycle period.

## a) Step 1: The Trend Estimation

Assuming that the time series under study has already seasonal adjusted. Then, the remaining in the series were the trend-cycle and irregular components. Trend is defined as the upward or downward movement observed in the data over several decades. This component represents smooth, gradual variations over long period of time. In the CE

<sup>1</sup> Trend is estimated from a centred 75-month (25- quarter) moving average. The beginning and end of the series being extrapolated.

<sup>&</sup>lt;sup>2</sup> Economic time series is assumed to have four main components: trend (T), cycle (C), seasonal (S) and irregular (I) and they are also assume in a multiplicative form which is  $Y_t = T_t \ x \ C_t \ x \ S_t \ x \ I_t$ 

method it is assumed that the trend-cycle components can be isolated. The issue is how does the trend is estimated?

There are numbers of methods in estimating the trend of economic time series. In growth cycle (GC) analysis, the long-term trend is estimated using the Phase Average Trend (PAT)<sup>3</sup> method. This method was widely used prior 1990s for example by the Center for International Business Cycle Research (CIBCR) as well as the Organization for Economic Cooperation and Development (OECD). In the recent development, the the filtering model such as the Hodrick-Prescott (HP) filter or Christiano Fidgerald (CF) filter are used as an alternative model to estimate long-term trend of the series. Nielsson and Gyomai (2011) compared the three methods: the PAT, the HP-filter and the CF-filter, and they found that the HP-filter and the CF-filter performs better than the PAT. The choice between HP-filter and CF-filter depend on the objective of analysis. They propose researchers to use the HP-filter if the early, clear and steady turning point signals are the priority and to use the CF filter if the analysis are sensitive to cumulative revisions.

The main objective of the CE model in this paper is to have clear and steady turning. Thus, as recommended by Nelson and Gyomai (2011), the HP-filter is the appropriate model to estimate the trend and cycle components. The HP filter decomposes time series (y<sub>t</sub>) into non-stationary trend (g<sub>t</sub>) and a stationary residual component (c<sub>t</sub>) or cyclical as follows:

$$y_t = g_t + c_t$$
 for  $t = 1,...,T$ . (1a)

The growth component should be smooth, so that the procedure recommended by Hodrick and Prescott (1997) is to minimize

$$\sum_{t=t}^{T} C_{t}^{2} + \lambda \sum_{t=t}^{T} [(g_{t} - g_{t-1}) - (g_{t-1} - g_{t-2})]^{2}$$
 (1b)

where the parameter  $\lambda$  is positive. If  $\lambda \to 0$ , the trend approximates the actual series  $y_t$  and if  $\lambda \to \infty$ , the trend become linear. According to Zarnowitz and Ozyildirim (2002) the estimate with larger  $\lambda$  is quite similar to PAT and if  $\lambda$  is increased by 7.5-fold will improve in the new H-P estimate of the "cyclical component". Although the recommended for monthly time series  $\lambda$  is 14,400 but for the purpose of this paper the size of  $\lambda$  will be increased by 7.5 fold as recommended by Zarnowitz and Ozyildirim.

## b. <u>Step2: The Cyclical Extraction</u>

The cyclical components is estimated by dividing the trend component (T) which is estimated in equation (1a) and (1b) to the equation (1c). Thus, the remaining components

<sup>&</sup>lt;sup>3</sup> PAT is deviations from a centred 75-month (25- quarter) moving average and from its extrapolations at the beginning and end of the series..

of the respective time series are the cyclical (C) and true irregular components (I') as in equation (1d).

$$Y_t = T_t * C_t * I_t$$
 (1c)

Thus, the remaining components of the respective time series are the cyclical (C) and true irregular components (I') as in equation (1d).

$$Y_t = C_t * I_t \tag{1d}$$

The true irregular (I), by definition should not be removed in the particular time series. However, for the purpose of analyzing the cyclical turning points, equation 1(d) is smoothed by applying HP filter with a very small lamda ( $\lambda$ ). Thus, the smoothed cyclical components is as in equation 1(e),

$$Z_t = C_t * I'_t \tag{1e}$$

where I' is the smoothed true irregular component

### c. Step3: Establish the Growth Cycle Reference Period (GCRP)

The purpose of GCRP is to check the conformity criteria and also to establish lead-lag analyses of each of time series under study. The RCP is estimated based on the cyclical components of Industrial Production Index which is derived from equation 1(a) to 1 (e). The highest and the lowest points in the cyclical component of the IPI respectively represent the "peak" and "trough" of the series or the GCRP.

### 3.2 Method 2: Causality Test Model (CT)

The cyclical extraction method relatively is time consuming. Researchers need to decompose time series under study step by step to eliminate its seasonal variation, trend-cycle and irregular components by using different types of statistical software and methodologies. As an alternative measures, researchers applied the causality test model such as the Unrestricted Vector Autoregressive (VAR) model, Restricted VAR or Vector Error Correction Model (VECM) or Toda-Yamamoto Model for business cycle indicator selection<sup>4</sup>.

This paper will apply simple granger causality test of two variables and their lags which is called the CT method. The reason of choosing this method is because of to compare the findings with the results of CE model.

<sup>&</sup>lt;sup>4</sup> The choice of the appropriate model to be applied is depend on the stationary test of time series.

Subjected to the stationary test results<sup>5</sup> which model to be applied whether the VAR or the VECM, the general causality model between the IPI and MS is as follows'

$$(IPI)_{t} = \alpha + \sum_{i=1}^{m} \beta_{i} (IPI)_{t-i} + \sum_{i=1}^{n} \tau_{j} (MS^{x})_{t-j} + \mu_{t}$$
 (2a)

$$(MS^{x})_{t} = \theta + \sum_{i=1}^{p} \phi_{i} (MS^{x})_{t-i} + \sum_{j=1}^{q} \overline{\omega}_{j} (IPI)_{t-j} + \mu_{t}$$
 (2b)

MS comprises of three sets of time series as explained in section 2. Based on the estimated OLS coefficients for the equations (2a) and (2b), four different hypotheses about the relationship between IPI & MS can be formulated:

- i) Unidirectional Granger-causality from MS to IPI. In this case the money supply increases the prediction of economic growth (represented by the IPI) but not vice versa. Thus,  $\sum_{j=1}^{n} \tau_{j} \neq 0$  and  $\sum_{j=1}^{q} \varpi_{j} = 0$
- ii) Unidirectional Granger-causality from IPI to MS. In this case the economic growth increases the prediction of the money supply but not vice versa. Thus,  $\sum_{i=1}^{n} \tau_{j} = 0$  and  $\sum_{i=1}^{q} \varpi_{j} \neq 0$
- iii) Bidirectional (or feedback) causality. In this case  $\sum_{j=1}^{n} \tau_{j} \neq 0$  and  $\sum_{j=1}^{q} \varpi_{j} \neq 0$  so the economic growth increases the prediction of money supply and vice versa.
- iv) Independence between IPI & MS. In this case there is no Granger causality in any direction, thus  $\sum_{i=1}^{n} \tau_{j} = 0$  and  $\sum_{i=1}^{q} \varpi_{j} = 0$

### 4. Findings

4.1 Findings from CE Method

a) The Growth Cycle Reference Period

Chart 1 below represents the dates of cyclical turning points of the IPI which is estimated from the equation 1(a) to 1(e). For the purpose of analysis, the cyclical "peak" and "though" of the IPI later called as the growth cycle reference period (GCRP).

<sup>&</sup>lt;sup>5</sup> The ADF and AIC is applied in this paper

115.0 10/9 110.0 105.0 100.0 95.0 90.0 85.0 Jan-94 Jan-95 Jan-96 Jan-97 Jan-98 Jan-99 Jan-00 -Jan-02 Jan-03 Jan-04 Jan-05 Jan-06 Jan-07 Jan-08 Jan-09 Jan-10 -Jan-12 Jan-93 Jan-11 Jan-92 Jan-01 Jan-

Chart 1: Growth Cycle of the Industrial Production Index Jan 90- Dec 2013

Shaded areas are the growth cycle recessions

Table 1: Growth Cycle Reference Period Jan 1990 to Dec 2013

	Date of turning points at			Expansion	Contraction	Full Cycle
Reference Cycle	Trough	Peak	Trough	(Month)	(Month)	(Month)
First Cycle	Dec-93	Oct-97	Dec-98	46	14	60
Second Cycle	Dec-98	Aug-00	Apr-02	20	20	40
Third Cycle	Apr-02	Feb-04	Jun-05	22	16	38
Forth Cycle	Jun-05	Jan-08	Jan-09	31	12	43
Fifth Cycle	Jan-09	May-10	May-11	16	12	28
Sixth Cycle	May-11	Feb-12	Feb-13	9	12	21
Average				24	14	38

It is observed that there were seven cyclical down-turns from January 1990 to December 2013. The full cycle (from peak to peak or from trough to trough) was estimated about thirty-eight months as in table 1. It was also observed that in the recent year the duration of full-cycle become much shorter as compared with in the 1990s. The significant dropped were for the expansion period which was from 46 months in 1990s to only 16 months and nine months respectively for fifth and sixth cycle lower than the average expansion (24 months).

# b) Lead-lag Tables Analysis

Table 2: Lead and Lag Table of Money Supply M1, M2 and M3, Jan 1990-Dec 2013

Reference Cycle		M1		M2		M3	
Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough
	Dec-93		Feb-93		Jul-93		Mar-93
Oct-97		Feb-97	-10	Dec-97	-5	Dec-97	-9
	Dec-98	-8	Dec-98	2	Oct-98	2	Jan-99
Aug-00		Jan-00	0	Dec-99	-2	Jul-99	1
	Apr-02	-7	Jul-01	-8	n.a	-13	Mar-03
Feb-04		Mar-04	-9	n.a		May-05	11
	Jun-05	1	Apr-06		May-04	15	May-06
Jan-08		Mar-08	10	Jul-08	-13	Jul-08	11
	Jan-09	2	Feb-09	6	n.a	6	May-09
May-10		n.a	1	n.a		Nov-09	4
	May-11		n.a		Feb-11	-6	Feb-11
Feb-12		Oct-11		Sep-12	-3	Sep-12	-3
	Feb-13	-4	Oct-12	7	n.a	7	Nov-13
			-4				9
Average lead or lag at peak			-3		2		2
(months)					2		2
_	or lag at trough	-4		-6			3
(months)			-	-0			3
Average lead of		-3		-2		3	
and trough (me	onths)			-			
Extra cycle		Feb 94 - Oct 95, Apr-02 - Mar 03		Feb 07 - Nov 07		Mar 94 - Apr 95, Feb 07 - Nov 07	
way 2010,	, way 2011	May 2010 a	and Feb 2013	n.a			
Percent of turning points		01	.7%	66.7%		100.0%	
conformity		71	. 7 /0	100.0%			

Table 2 is the lead-lag analysis of turning points for money supply M1, M2 and M3 compared with the turning points of GCRP. The dates for GCRP were taken from Chart 1 while the turning point dates of M1, M2 and M3 respectively were taken from Chart 2, Chart3 and Chart 4 as in appendix 1. In the perspective of "conformity", the M3 is better than M1 or M2. The M3 "conform" with all turning points of GCRP while M1 represents 91.7 per cent and M2 only 66.7 per cent. In term of predictive measure, the M1 is better than M2 or M3 since M1 lead the overall turning points of GCRP by three months compared with only two months for M2. The M3, however, indicated that it is a lagging series. Based on these analyses, M1 is recommended as a potential BCI because it fulfils the "conformity" criteria and also the "leading" criteria.

### 4.2 Findings of CT Model

## a) The Stationary Test Results

The results of stationary test showed that all series were stationary at the first difference (Table 3). The findings recommend us to apply the Johansen

cointegration test to examine the relationship between the IPI and money supply

Table 3: The Stationary Test Results of IPI, M1, M2 and M3

No	Series Name	Level		First Difference		
		Constant	Constant and Trend	Constant	Constant and Trend	
1	Industrial Production Index	0.7416	0.4275	0.0000*** (2)	0.0000 *** (2)	
2	Money Supply M1	1.0000	1.000	0.1794	0.0032** (12)	
3	Money Supply M2	1.0000	0.9988	0.0570 *(7)	0.0013 ***(9)	
4	Money Supply M3	1.0000	1.0000	0.0385**(7)	0.0000*** (3)	

Note:

\*\*\*, \*\* and \* is the significant level at 1%, 5% and 10% respectively. Number in bracket is the optimal lag based on Akaike Info Critirion (AIC).

# b) The CT Findings

Table 4: The Causality Test Results of the IPI,M1. M2 and M3

No.	Varibles	Duration of	Cointegra	tion Test	Causality	Test
		Data	Methods	Results	Methods	Results
1	Industrial Production index vs Money Supply M1		Johansen Cointegration Test	Exist long-run relationship	VEC Granger Causality/Block Exogeneity Wald Tests	Bi directional relationship netween IPI and M1
2	Industrial Production index vs Money Supply M2		Johansen Cointegration Test	Ü	<i>U</i> ,	No causality exist between IPI and Money Supply-M2
3	Industrial Production index vs Money Supply M3		Johansen Cointegration Test	Exist long-run relationship	<i>U</i> ,	No causality exist between IPI and Money Supply-M3

Base on Johansen cointegration test it can be concluded that the IPI has a long run relationship with money supply M1, M2 and M3 (table 4). However, the causality test showed that the IPI has no relationship with M2 and M3 in the short run. These finding suggests that the M1 and IPI has a bi-directional which imply the M1 is a candidate for BCI as a "coincident series" which is contradict with the findings using the CE model.

### 4.3 Comparison the findings of the CE and CT

The findings of the CE and the CT models as describe in section 4.1 and 4.2 are illustrated in table 5. To simplify the findings, the CT model suggests the money supply-M1 is a "coincident series" while the CE model recommends as a "leading series" which is contradicting between them. The conformity of turning points of the M1 is 91.7 per cent which also suggested the M1 as the best potential component of BCI.

The CE model recommends M2 as a leading. However, the conformity criterion indicates this indicator was not recommended to be selected as a candidate for BCI. M2 only conforms 66.7 percents of GCRP. The CT model also rejects M2 as a potential candidate for BCI. The CT model indicates that there is no causality exists between IPI and M2.

Table 5: Summary of the CT and CE Findings for IPI, M1, M2 and M3

		Cas	uality Test (CT) Mo	del Cyclical Extraction (CE) Model			odel
	Variables Tested			Potential to be	Turning Points	Percent of Turning	Potential to be
		Model	Results	selected as BCI	Comparison	Points Conformity	selected as BCI
1	IPI and Money	VEC Granger	Bi-directional	Coincident	IPI Reference Cycle	91.7	Leading
	Supply M1	Causality/Block	relationship exist		and Money Supply M1		
		Exogeneity Wald	between IPI and				
		Tests	Money Supply M1				
2	IPI Money Supply	VEC Granger	No causality exist	Rejected	IPI Reference Cycle	66.7	Leading
	M2	Causality/Block	between IPI and		and Money Supply M2		
		Exogeneity Wald	Money Supply M2				
		Tests					
3	IPI Money Supply	VEC Granger	No causality exist	Rejected	IPI Reference Cycle	100	Lagging
	M3	Causality/Block	between IPI and		and Money Supply M3		
		Exogeneity Wald	Money Supply M3				
		Tests					

The CE model recommends M3 as a lagging series. However, the CT model was not recommend M3 to be selected as a candidate for BCI. The CT model indicates that there is no causality exists between IPI and M3.

The above findings propose that CE model is better than the CT model. However, both models can be used simultaneously especially in the case of to examine a bulk of time series. As mention earlier that the CE models is time consuming since the model apply various statistical packages decompose time series step by step. If the CT model is applied prior to the CE models some of the "unnecessary" time series can be neglected. Thus, the CE model only examined the time series that has "selected" by the CT model.

### Conclusion

The main objective of this paper is to evaluate the outcome of CE and CT models especially in determining the appropriate indicators for BCI. The IPI is used as a benchmark indicator while the money supply M1, M2 and M3 as the tested indicators.

Overall findings suggest that the outcome of the CE and CT models were very different especially in classifying the indicators as a "leading", "coincident" or "lagging" series. The CE model recommended M1 as a "leading" while the CT suggests M1 as a "coincident" series. For M2, the CE recommended the series as a "leading", however, the conformity criterion does not suggest M2 be selected. The CT model also rejects the

M2 as the result indicates that there is no causality exists between IPI and M2. For the M3, the CE model, recommends as a "lagging" series. However, the CT indicates that there is no causality exists between IPI and M3.

The findings also suggest that the CE and CT model can be used simultaneously in business cycle analysis. The CT helps to identify the relationship of each of time series while the CT provide extra information on "conformity" criterion as well as the magnitude of predictive measure.

T 12/93 T 6/05 T 1/09 P 2/04 10/97 4/02 1/08 5/10 5/11 2/12 2/13 12/98 8/00 120.0 115.0 110.0 4/02 105.0 10/11 100.0 95.0 7/01 3/03 10/12 90.0 85.0 80.0 75.0 Jan-93 Jan-97 Jan-99 Jan-00 Jan-02 Jan-03 Jan-06 Jan-08 Jan-94 Jan-04 Jan-05 Jan-07 Jan-09

Chart 2: Growth Cycle of Monet Supply (M1), Jan 90- Dec 2013

Shaded areas are the growth cycle recessions

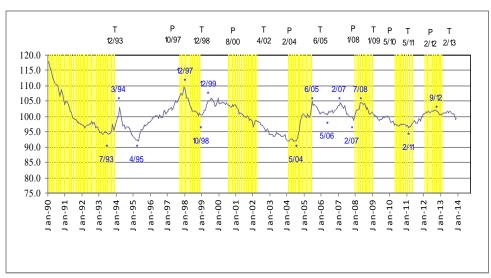
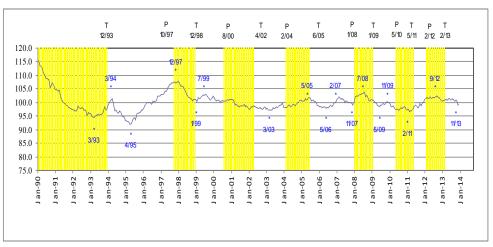


Chart 3: Growth Cycle of Monet Supply (M2), Jan 90- Dec 2013

Shaded areas are the growth cycle recessions

Chart 4: Growth Cycle of Monet Supply (M3), Jan 90- Dec 2013



Shaded areas are the growth cycle recessions

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